Math 10B with Professor Stankova
Quiz 5; Tuesday, 2/26/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE** False If $A \subset B$, then $P(B|A) = 1$ (assuming all quantities are well defined).

Solution: Writing the formula, we get $P(A \cap B) = P(A)$.

2. True **FALSE** If $P(A), P(B) \neq 0$, then $P(A|B) = P(B|A)$.

Solution: See the quiz problem for a counter-example.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Suppose a new cancer test has a 90% chance of correctly identifying that a sick patient has cancer and a 10% chance of incorrectly identifying that a healthy patient has cancer. Assume that 20% of the population has this form of cancer.

   (a) (2 points) Let A be the event that a random person has cancer and B being the event that a person tests positive for cancer. Write the probabilities you are given in terms of A and B.

   Solution: Then we are told that $P(B|A) = \frac{9}{10}, P(B|\bar{A}) = \frac{1}{10}, P(A) = \frac{1}{5}$.

   (b) (3 points) What is the probability that the test says a random person has cancer?

   Solution: We are asked to calculate $P(B)$ and we get

$$P(B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$
$$= \frac{9}{10}\frac{1}{5} + \frac{1}{10}\frac{4}{5} = \frac{13}{50}.$$

   (c) (5 points) What is the probability that a person who tests positive has cancer?

Solution: We are asking for $P(A|B)$ and we use Bayes theorem to get

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$
$$= \frac{\frac{9}{10}\frac{1}{5}}{\frac{13}{50}} = \frac{9}{13}.$$